

Optimized BiLSTM-Based Short-Term Prediction of BeiDou Satellite Clock Bias for High-Precision GNSS Applications

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Abstract— Accurate satellite clock bias prediction is essential for improving the positioning accuracy and reliability of Global Navigation Satellite Systems (GNSS). However, the nonlinear, time-varying, and complex temporal characteristics of BeiDou satellite clock bias make accurate forecasting a challenging task for conventional prediction approaches. This study proposes an optimized Bidirectional Long Short-Term Memory (BiLSTM)-based deep learning framework for short-term prediction of BeiDou Navigation Satellite System (BDS) satellite clock bias. The proposed model is evaluated against traditional prediction methods, including Quadratic Polynomial (QP), Periodic (SA), Grey Model GM (1,1), and conventional Long Short-Term Memory (LSTM) networks. Experiments are conducted using high-precision BDS-3 satellite clock bias data obtained from the International GNSS Service (IGS), covering different satellite types, including Medium Earth Orbit (MEO), Inclined Geosynchronous Orbit (IGSO), and Geostationary Earth Orbit (GEO) satellites with different atomic clock configurations. The model performance is investigated under single-day and multi-day prediction scenarios with forecasting intervals of 1-hour, 6-hour, and 12-hour. Prediction accuracy is evaluated using Root Mean Square Error (RMSE), error range, and standard deviation analysis. The experimental results demonstrate that the optimized BiLSTM model consistently achieves superior prediction accuracy and more stable error characteristics compared with traditional and unidirectional deep learning models. The proposed approach effectively captures complex temporal dependencies in satellite clock bias sequences and provides a reliable solution for high-precision GNSS timing and navigation applications.

Keywords— BeiDou Navigation Satellite System; Satellite Clock Bias; BiLSTM; Deep Learning; GNSS; Time Series Prediction

I. INTRODUCTION

Global Navigation Satellite Systems (GNSS) are playing a crucial role, having established a network of essential elements in modern positioning, navigation and timing (PNT) services, including for various modes of transport, autonomous navigation, geospatial surveying, communication networks and safety-critical systems [1]. The

major GNSS constellations such as the Global Positioning System (GPS) [2], Galileo, and the BeiDou Navigation Satellite System (BDS) [3], offer continuous positioning and timing services via signals broadcast from satellites that incorporate highly accurate onboard atomic clock devices. The BeiDou Navigation Satellite System is one of these systems to be a global one which has included a hybrid constellation of Medium Earth Orbit (MEO), Inclined Geosynchronous Orbit (IGSO), and Geostationary Earth Orbit (GEO) satellites to accomplish its goals. Satellite clock performance is an important factor in the accuracy and reliability of the GPS/GLONASS (GNSS) positioning. Satellite clocks are not too far wrong, as even the smallest discrepancy in the timing would cause large positioning errors, since the propagated distance between satellites and receivers is proportional to the delay time [4].

Satellite clock bias, defined as the difference between the onboard satellite clock time and a reference time standard, is one of the major error sources affecting GNSS positioning accuracy [5]. Although modern GNSS satellites are equipped with highly stable atomic clocks, including rubidium and hydrogen maser clocks, clock deviations caused by frequency drift, environmental variations, satellite dynamics, and hardware characteristics cannot be completely eliminated. These factors result in clock bias sequences with nonlinear, time-dependent, and irregular characteristics, making accurate prediction a challenging task. In GNSS positioning, the pseudo range measurement between a satellite and receiver can be expressed as equation 1:

$$\rho = c(t_r - t_s) + I + T + \epsilon \quad (1)$$

Here is equation 1 ρ represents the measured pseudorange, c denotes the speed of light, t_r and t_s represent receiver and satellite signal transmission times, respectively, while I , T , and ϵ indicate ionospheric delay, tropospheric delay, and measurement errors. Since satellite clock bias directly affects satellite time estimation, inaccurate clock prediction can propagate into positioning errors [6]. Therefore, reliable



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short-term prediction of satellite clock bias is essential for improving the accuracy and stability of GNSS applications.

The traditional methods are popularly used for the satellite clock bias prediction because of simplicity and computational ability. The Quadratic Polynomial (QP) model tries to model the clock differences based on historical observations of clock biases with polynomial models; Periodic (Semi-Parametric) models include polynomial periodic parameters due to the satellites' orbital movement and due to the dynamics of the system. In the same way, prediction problems with a small amount of data or with data uncertainty have been solved by the Grey Model GM (1,1). These mathematical methods can be used to obtain acceptable results for short time periods and relatively stable clock variations; however, such methods are not as effective when complex and nonlinear clock variations are present, and for longer prediction periods. The satellite clock bias is dynamically time dependent and randomly varying such that traditional models are insufficient to capture the time series' attributes [7].

Unlike conventional LSTM, which processes the input sequence in a single temporal direction, BiLSTM processes the available input window in both forward and reverse directions. For satellite clock-bias forecasting, the bidirectional operation is applied only within the historical observation window available before the forecast origin. Thus, the backward pass does not provide access to future clock-bias observations beyond the prediction time. Instead, it enables the model to encode temporal relationships among observations within the historical window from both directions, potentially providing a richer representation of the observed temporal dynamics. This property may be beneficial for satellite clock-bias sequences that exhibit nonlinear and time-varying behavior.

Deep learning has proved to be powerful in modeling nonlinear sequential data and with the advent of Artificial Intelligence, it is now seeing a lot of advancements. Recurrent Neural Networks (RNNs) allowed for the processing of time dependent information, however they had drawbacks in terms of vanishing gradients and inability to learn long term dependency [8]. In order to tackle these issues, Long Short-Term Memory (LSTM) networks were introduced which have built-in memory cells and gating mechanisms, allowing for the successful learning of long-range temporal patterns. LTMs are able to model complex dependency and have been successfully used for different time series prediction tasks. But the classical LSM integrates the sequential information in only one direction, therefore it cannot make full use of the total temporal context [9]. Bidirectional Long Short-Term Memory (BiLSTM) networks are similar to LTMs, but use forward and backward processing layers to learn from past and future contexts of the sequence to be classified. This bi-directional architecture not only helps to better represent the features but also makes BiLSTM a very promising model for problems in the field of nonlinear time-series forecasting, like satellite clock bias prediction [10].

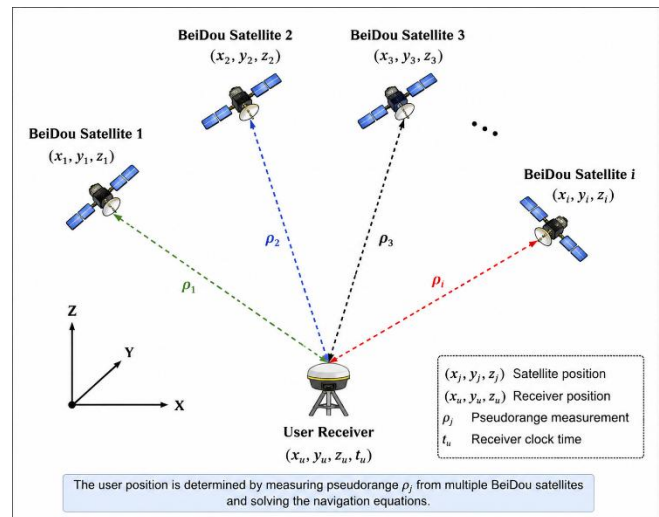


Figure 1: Geometry of BeiDou GNSS Positioning Based on Pseudo range Measurements.

This figure illustrates the basic positioning principle of the BeiDou Navigation Satellite System (BDS) based on pseudorange measurements. Multiple BeiDou satellites transmit signals with their known positions (x_j, y_j, z_j) , while the user receiver estimates its location by measuring the signal travel time from each satellite. The measured pseudorange (ρ_j) represents the distance between the satellite and receiver and is used to solve the navigation equations. By combining measurements from multiple satellites, the receiver can determine its three-dimensional position (x_u, y_u, z_u) along with clock-related errors. The accuracy of GNSS positioning strongly depends on precise satellite clock information because clock bias directly affects distance estimation and positioning reliability [11].

Some tests have been done in the field of prediction models for the time-series analysis related to GNSS. The previous studies on the BeiDou system mainly involve constellation design, the positioning performance and the development of the satellite system while advanced satellite clock bias prediction methods have been ignored [12]. Researches have been done on satellite clock errors, which concluded that clock bias is an important factor of the positioning errors in the GNSS; however, they concentrated on analysis of the clock errors rather than the creation of intelligent prediction models. The traditional model like QP model and GM (1,1) model had some obvious computational advantages but in terms of capturing nonlinear clock variations, they could not do well. New studies on the use of deep learning techniques, such as LSTM-based models, reported improvement in the accuracy of the prediction of GNSS time series. But, application of BeiDou satellite clock bias prediction using bidirectional learning strategies is still relatively unexplored. In addition, the impact of various satellite types such as MEO, IGSO and GEO satellites with varying atomic clocks needs to be studied in order to build up a comprehensive and general prediction model [13].

Considering these shortcomings, an advanced prediction model to effectively learn the nonlinear temporal patterns of BeiDou satellite clock bias sequences still exist. Major research gaps that this study will fill: Being able to only apply the BiLSTM-based approaches to short-term prediction of BeiDou satellite clock bias, and not be able to

comprehensively compare with other traditional mathematical models and other conventional deep learning methods [14]. In view of these difficulties, the present study suggests an optimum BiLSTM-based prediction framework along with the example of the forecasting of satellite clock bias with high precision in short term. The high precision BDS-3 satellite clock bias data provided by the International GNSS Service (IGS) are utilized to validate the proposed approach along with Quadratic Polynomial (QP), Periodic (SA), Grey Model GM (1,1) and standard LSTM models. The proposed method is tested for various prediction situations, such as both short and extended time horizon prediction (1 hour, 6 hours, 12 hours prediction), as well as single-day and multi-day prediction experiments [15].

The key contributions of this study are: (1) A BiLSTM-based optimized framework is proposed to improve the clock bias prediction performance of the BeiDou satellite system in short time horizon; (2) Extensive comparisons are conducted to validate the effectiveness of the proposed model over the traditional and the deep learning-based ones; and (3) The prediction performance is tested under different satellite types and clock configuration on various time horizons to test the stability and generalization ability of the model. Results of this study illustrate an efficient deep learning approach for enhancing the timing prediction of satellites and for further applications of high-precision GNSS.

This paper continues with the following. The proposed methodology was discussed in Section 2, and the mathematical models and optimized BiLSTM architecture were presented. The description and experimental setup of the data set and evaluation criteria are given in Section 3. In Section 4, experimental results and a comparative analysis are discussed. Last but not least, the study concludes and indicates the directions in which future research should continue in Section 5.

II. METHODOLOGY

In this study, an optimized framework called Bidirectional Long Short-Term Memory (BiLSTM) for short-term prediction of BeiDou satellite clock bias is proposed. The methodology is divided into 5 major steps: Satellite clock bias data acquisition, preprocessing of time-series observations, establishment of conventional and deep learning prediction models, establishment of an optimized BiLSTM model and performance evaluation. Traditional mathematical prediction models and deep learning prediction models are implemented to analyze the effectiveness of the proposed approach in all the conditions with different forecasting scenarios.

A. Traditional Prediction Models

For the sake of doing a comprehensive comparison, three classic prediction methods Quadratic Polynomial (QP), Periodic (Semi-Parametric) model, and Grey Model GM (1,1) are studied. Because of their simple architecture and low computation; these have been widely used to predict the satellite clock bias [16]. But, they still have limited capability for capturing nonlinear and nonstationary satellite clock bias features [17].

1- Quadratic Polynomial (QP) Model

The Quadratic Polynomial model assumes that satellite clock bias changes follow a smooth temporal trend that can be approximated using a second-order polynomial function [18]. The mathematical representation of the QP model is expressed as:

$$\Delta t(t) = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 \quad (2)$$

where $\Delta t(t)$ represents the satellite clock bias at time t , a_0 denotes the clock offset, a_1 represents the frequency offset, and a_2 indicates the frequency drift coefficient. The parameters are estimated using historical clock bias observations. Although the QP model performs effectively for short-term smooth variations, its accuracy decreases when the clock bias contains complex nonlinear fluctuations over longer prediction intervals.

2- Periodic (Semi-Parametric) Model

Satellite clock bias often contains periodic characteristics caused by orbital motion and system dynamics. Therefore, the Periodic (SA) model extends the polynomial approach by incorporating sinusoidal components to represent periodic variations. The model can be expressed as:

$$\Delta t(t) = a_0 + a_1(t - t_0) + \sum_{i=1}^n [b_i \sin(\omega_i t) + c_i \cos(\omega_i t)] \quad (3)$$

where b_i and c_i represent the amplitudes of periodic components, ω_i denotes angular frequency, and n represents the number of periodic terms. By combining trend and periodic information, this model provides improved prediction capability compared with pure polynomial fitting. However, it remains dependent on accurate identification of periodic patterns and has difficulty modeling irregular nonlinear variations.

3- Grey Model GM(1,1)

The Grey Model GM(1,1) is a mathematical prediction method designed for systems with limited or uncertain information. It transforms the original time-series sequence through an accumulated generating operation (AGO) and constructs a first-order differential equation to estimate future values. The predicted sequence is represented as:

$$x^{(0)}(k+1) = (1 - e^a) \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} \quad (4)$$

where a is the development coefficient and b is the grey input parameter. Although GM(1,1) is computationally efficient and suitable for small datasets, it assumes relatively regular trends and cannot effectively capture the nonlinear temporal dependencies present in satellite clock bias sequences.

B. Long Short-Term Memory (LSTM) Network

LSTM is a recurrent neural network architecture designed to overcome the limitations of conventional RNNs in learning long-term sequential dependencies [19]. The LSTM structure contains memory cells and three major gates: input gate, forget gate, and output gate. These gates control information flow and allow the network to selectively retain or discard information during sequence processing [20]. The input gate determines the amount of new information added to the memory cell:

$$i_t = \sigma(W_i[h_{t-1}, x_t] + b_i) \quad (5)$$

The forget gate controls previously stored information:

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f) \quad (6)$$

The output gate generates the hidden state:

$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o) \quad (7)$$

where x_t represents the input sequence at time t , h_{t-1} denotes the previous hidden state, W represents weight matrices, and b represents bias parameters. Although LSTM provides strong capability for time-series prediction, it processes information in only one direction, which limits its ability to utilize complete temporal context.

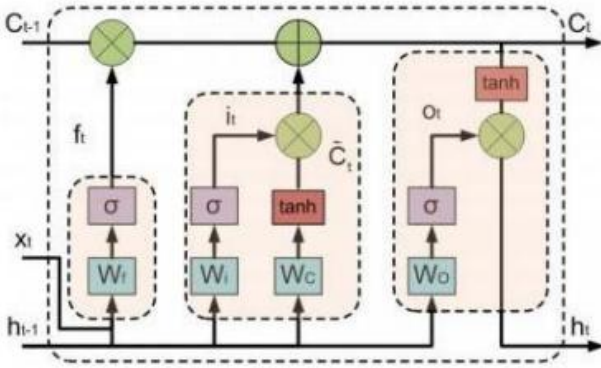


Figure 2: Structure diagram of LSTM network

In the figure w , the input of the current moment is x_t , the output of the current moment h_t is the output of the forgetting gate f_t , the output of the input gate of the current moment is i_t , and the output of the output gate of the current moment is o_t . The cell state is sent to the "tanh" layer, outputting a probability value between -1 and 1 , multiplying the probability value by the output of the S-curve layer. In the figure, W is the weight matrix, b is the bias vector, σ and $\tanh$ is the S-type activation function and the hyperbolic tangent activation function, respectively [21].

C. Optimized BiLSTM Prediction Model

To overcome the limitation of unidirectional temporal learning, this study employs an optimized Bidirectional Long Short-Term Memory (BiLSTM) model. Unlike conventional LSTM, BiLSTM consists of two parallel LSTM layers that process the input sequence in forward and backward directions [22]. This structure enables the model to process the historical input sequence in both forward and reverse directions. In the forecasting setting considered here, the input sequence contains only observations available before the forecast origin; therefore, the reverse pass does not use future observations beyond the prediction time. The forward LSTM hidden state is calculated as:

$$\vec{h}_t = LSTM_f(x_t, \vec{h}_{t-1}) \quad (8)$$

while the backward LSTM processes the sequence in reverse direction:

$$\overleftarrow{h}_t = LSTM_b(x_t, \overleftarrow{h}_{t+1}) \quad (9)$$

The two hidden representations are combined to generate the final temporal feature representation:

$$h_t = [\vec{h}_t; \overleftarrow{h}_t] \quad (10)$$

where h_t contains information from both temporal directions. Through this bidirectional learning mechanism, the proposed model can better capture nonlinear relationships and complex temporal patterns within BeiDou satellite clock bias sequences. This makes BiLSTM more suitable for accurate short-term satellite clock bias prediction compared with traditional approaches and standard LSTM models.

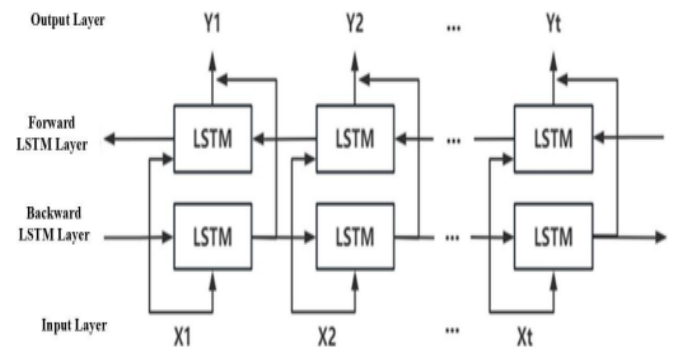


Figure 3: BiLSTM structure diagram

Figure 3 illustrates the forecasting architecture in which $X_1, \dots, X_t$ represent the historical clock-bias observations available up to the forecast origin, while Y_{t+h} represents the future clock-bias value at the selected forecasting horizon. The forward and backward LSTM layers process only the historical input window in opposite directions. The backward operation therefore does not access observations occurring

after t . The resulting hidden representations are combined and passed to the prediction layer to estimate the future clock-bias value. On this basis, this hidden state is used to generate the predicted clock difference data for the corresponding moment [23].

III. DATASET DESCRIPTION AND EXPERIMENTAL SETUP

High accuracy BeiDou Navigation Satellite System (BDS-3) satellite clock bias data is used for the experimental test of the proposed optimized BiLSTM model. The data for this study is from the International GNSS Service (IGS), a service offering high accuracy satellite orbit and clock data for GNSS research and applications. The chosen data set has highly accurate satellite clock bias observations in terms of time accuracy, which allows reliable assessment of the short-term prediction quality. In this work, different types of satellites of the Beidou system, such as Medium Earth Orbit (MEO), Inclined Geosynchronous Orbit (IGSO) and Geostationary Earth Orbit (GEO) satellites are taken into account. In addition, multiple onboard atomic clock configuration (such as rubidium on-board atomic clock and hydrogen clock) is also proposed to investigate the robustness and generalization capability of the proposed model across different satellite timing properties [24].

The satellite clock bias observations represent sequential time-series data, with historical clock bias data used as input data when predicting clock bias behavior. The prediction model is trained to learn temporal dependencies and hidden patterns in the historical observations since the satellite clock bias is also time-varying and nonlinear. The experimental framework aims at assessing the prediction potential of various models for different forecasting scenarios [25]. This general experimental procedure involves data preparation, model training, clock bias prediction for future measurements and performance evaluation.

For the sake of comparison, the prediction experiments are done under the single day and multi day forecast scenarios. In the 1-day prediction scenario, various forecasting horizons are analyzed such as 6-hour, 12-hour and 1-hour prediction periods. The various time horizons have been chosen to study the impact of the time horizon increase on the model's precision and stability. Moreover, the multi-day prediction experiments are performed to further test the ability of the proposed BiLSTM model to model the outer temporal changes in satellite clock bias sequences [26].

The performance of the proposed optimized BiLSTM model is verified and compared with four existing prediction models such as Quadratic Polynomial (QP), Periodic (SA), Grey Model GM(1,1), and conventional Long Short-Term Memory (LSTM). The QP, SA and GM(1,1) are traditional mathematical forecasting methods, and LSTM are the baseline methods of sequential prediction based on deep learning. As part of comparative evaluation, the same work is conducted to prove effectiveness of effectiveness of BTWL for increasing satellite clock bias prediction accuracy [27].

A. Evaluation Criteria

The prediction performance of QP, Periodic (SA), GM(1,1), LSTM, and BiLSTM is evaluated using three complementary criteria: RMSE, error range, and standard deviation (STD). RMSE is used as the primary accuracy

metric, where a lower RMSE indicates a smaller overall difference between predicted and observed satellite clock bias values. The error range, defined as the difference between the maximum and minimum prediction errors, is used to characterize the spread of prediction errors. STD is used to assess the consistency and stability of the prediction errors, with a lower STD indicating less variation around the mean error. Therefore, models are compared primarily according to RMSE, while error range and STD provide additional information about prediction stability and consistency.

The prediction capability of all the models is quantified by Root Mean Square Error (RMSE), error range and standard deviation [28]. It is decided to choose RMSE as primary evaluation index due to that it can reflect the bias value differences between prediction result and actual value, and give greater attention to larger error in prediction result. The RMSE value is calculated as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (11)$$

where y_i represents the actual clock bias observation, $\hat{y}_i$ represents the predicted value generated by the model, and n denotes the total number of observations. A lower RMSE value indicates better prediction accuracy. In addition to RMSE, error distribution characteristics are analyzed through error range and standard deviation to evaluate the stability and reliability of each prediction method.

IV. RESULTS AND DISCUSSION

A. Single-Day 1-Hour Clock Bias Prediction

The prediction experiment was carried out for one day based on accurate satellite clock bias data of BeiDou on February 14, 2023. For 1-hour forecasting, the performance of five models to be predicted was compared: Quadratic Polynomial (QP), Periodic (SA), Grey Model GM(1,1), Long Short-Term Memory (LSTM), and the optimized BiLSTM model proposed in this paper. The Root Mean Square Error (RMSE) in nanoseconds (ns) was used to estimate the accuracy of the prediction.

Table 1: RMSE values (ns) of different models for 1-hour BDS satellite clock bias prediction

PRN	QP	SA	GM	LSTM	BiLSTM
C19	0.183	0.149	0.146	0.158	0.038
C20	0.105	0.135	0.085	0.187	0.080
C21	1.304	0.069	0.345	0.069	0.028
C22	1.282	0.116	0.119	0.128	0.088
C23	2.157	0.137	0.168	0.137	0.030
C24	2.166	0.097	0.063	0.088	0.031
C25	0.175	0.041	0.234	0.048	0.049
C26	5.014	0.076	0.051	0.036	0.047
C27	0.076	0.012	0.103	0.029	0.016
C28	0.043	0.050	0.136	0.050	0.107
C29	0.072	0.049	0.321	0.035	0.078
C30	0.171	0.060	0.035	0.032	0.036
C31	1.709	0.074	0.140	0.074	0.039
C32	0.800	0.066	0.079	0.066	0.028
C33	0.163	0.036	0.072	0.039	0.059

C34	0.172	0.120	0.108	0.027	0.072
C35	1.332	0.013	0.078	0.013	0.013
C36	1.464	0.023	0.069	0.023	0.037
C38	0.238	0.052	0.024	0.021	0.028
C39	0.071	0.023	0.148	0.031	0.013
C40	0.196	0.048	0.042	0.035	0.048
C41	2.199	0.163	0.080	0.205	0.090
C42	1.029	0.093	0.070	0.087	0.126
C43	0.021	0.038	0.064	0.009	0.037
C44	1.820	0.069	0.074	0.105	0.063
C45	4.195	0.029	0.039	0.023	0.027
C46	0.954	0.045	0.064	0.019	0.059
C59	1.482	0.138	0.487	0.026	0.072
C60	0.619	0.057	0.046	0.070	0.050

The results show that the optimized BiLSTM model obtains the minimum RMSE for most BeiDou satellites, which means it has a better prediction ability than the traditional model and standard LSTM model. Polynomial fitting is invalid to model the nonlinear variations in the satellite clock bias sequences (some of the satellites showing larger errors include C23, C24, and C26) which explains such relatively large errors. The SA and GM models yielded better prediction results than QP but with varying accuracy among different satellites.

The LSTM network can better model temporal relations between successive clock offsets and thus outperforms traditional models. But it is a one-way structure, so it is unable to get all temporal information. BiLSTM, however, takes the sequence in forward and backward directions, which makes for bidirectional encoding of the historical input

window. Thus, the prediction errors of BiLSTM are lower and the performance is more stable for different satellite conditions.

Compared with the conventional LSTM model, the optimized BiLSTM reduced the mean RMSE across the 29 evaluated BeiDou satellites by 20.37% for the 1-hour horizon, 32.98% for the 3-hour horizon, 46.29% for the 6-hour horizon, and 49.28% for the 12-hour horizon. In the multi-day experiment, the mean RMSE reduction was 26.24%. These results provide a quantitative measure of the improvement achieved by bidirectional temporal processing.

Table 2: Improvement of BiLSTM over LSTM

Forecast horizon	Mean LSTM RMSE (ns)	Mean BiLSTM RMSE (ns)	RMSE reduction
1-hour	0.0645	0.0513	20.37%
3-hour	0.1769	0.1186	32.98%
6-hour	0.3129	0.1681	46.29%
12-hour	0.5750	0.2917	49.28%
Multi-day	0.3854	0.2843	26.24%

An error distribution analysis also shows the benefits of the BiLSTM. From the mean difference in the RMSE histogram, it is clear that BiLSTM has smaller differences in RMSE whereas the traditional models have wide difference in RMSE. Moreover, BiLSTM's error interval is narrow indicating that the proposed model not only can effectively improve the accuracy but can also make more reliable and stable predictions.

Figure 4 shows the logarithmic distribution of RMSEs of the various forecasting models in their 1-hour scenario.

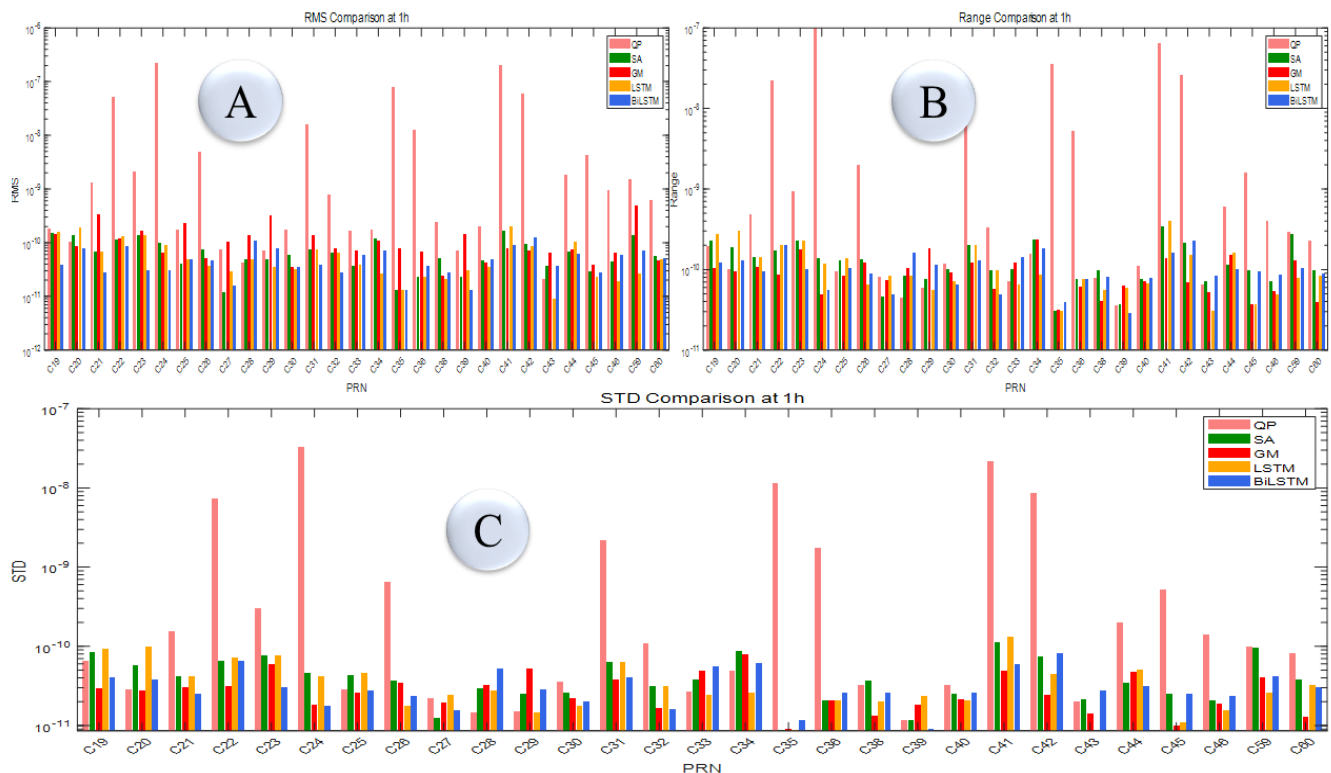


Figure 4: RMSE logarithmic histogram of BDS clock difference prediction for 1 hour with different models, : (A) RMSE comparison, (B) Standard deviation (STD) comparison, and (C) Error range comparison.

B. Forecast of 3h difference in a single day

It can be seen that different prediction models (QP, SA, GM, LSTM, BiLSTM) have a comparison to measure their performance on the 3-hour forecasting horizon, RMS, standard deviation (STD), range-based Error distributions, respectively, as shown in figure 5. These numbers give a pictorial view of the different distribution of prediction errors among the various models and satellites making it easier to interpret them as compared to numerical tables. The total error values in the 3-hour scenario are significantly more than those in the case of 1 hours as this shows the heightened complexity of medium-term forecasting.

Not surprising as prediction error can build up with the increase in prediction horizons. Nonetheless, the BiLSTM model is still highly performing in spite of this growth. The three figures have concentrated distributions of the errors in BiLSTM in the low side giving a constant lesser error in prediction amongst the various satellites. For traditional technologies, such as QP and GM, however, the errors follow a normal distribution, and therefore are more scattered and also less predictable in terms of performance. In certain occasions, the SA model is better than the QP but it is nevertheless inconsistent between satellites. The LSTM model shows a better performance with its capability to represent the temporal dependencies of the performance which still lacks the bidirectional learning (BiLSTM) feature thus leading to a relatively high error. Overall, the visualization images primarily capture the numerical results presented in Table 3, and visualize well the fact that in prediction tasks, BiLSTM is more likely to generate greater accuracy than is steady in instances of medium-term (3-hour) behavior.

Table 3: RMSE value (ns) of the 3h result of the BDS satellite clock difference forecast

PRN	QP	SA	GM(1,1)	LSTM	BiLSTM
C19	0.339	0.256	0.160	0.418	0.053
C20	0.086	0.363	0.061	0.478	0.139
C21	1.732	0.098	0.311	0.107	0.086
C22	3.312	0.397	0.126	0.442	0.185
C23	3.401	0.425	0.382	0.425	0.221
C24	3.977	0.219	0.071	0.121	0.157
C25	0.166	0.077	0.254	0.062	0.050
C26	7.709	0.166	0.128	0.066	0.087
C27	0.046	0.042	0.074	0.073	0.027
C28	0.097	0.155	0.341	0.171	0.164
C29	0.133	0.048	0.471	0.054	0.068
C30	0.199	0.078	0.075	0.060	0.068
C31	2.037	0.286	0.304	0.350	0.148
C32	1.317	0.271	0.204	0.259	0.092
C33	0.159	0.047	0.411	0.053	0.157
C34	0.132	0.096	0.276	0.228	0.071
C35	1.026	0.027	0.071	0.027	0.039
C36	1.858	0.027	0.065	0.036	0.072
C38	0.349	0.174	0.056	0.083	0.108
C39	0.051	0.070	0.132	0.159	0.077
C40	0.306	0.077	0.068	0.063	0.112
C41	2.679	0.409	0.168	0.561	0.284
C42	0.915	0.164	0.171	0.218	0.361
C43	0.041	0.072	0.065	0.057	0.067
C44	2.754	0.122	0.155	0.207	0.097
C45	1.289	0.059	0.045	0.046	0.076
C46	1.636	0.035	0.138	0.038	0.053
C59	1.788	0.246	0.327	0.114	0.069
C60	0.963	0.246	0.093	0.155	0.251

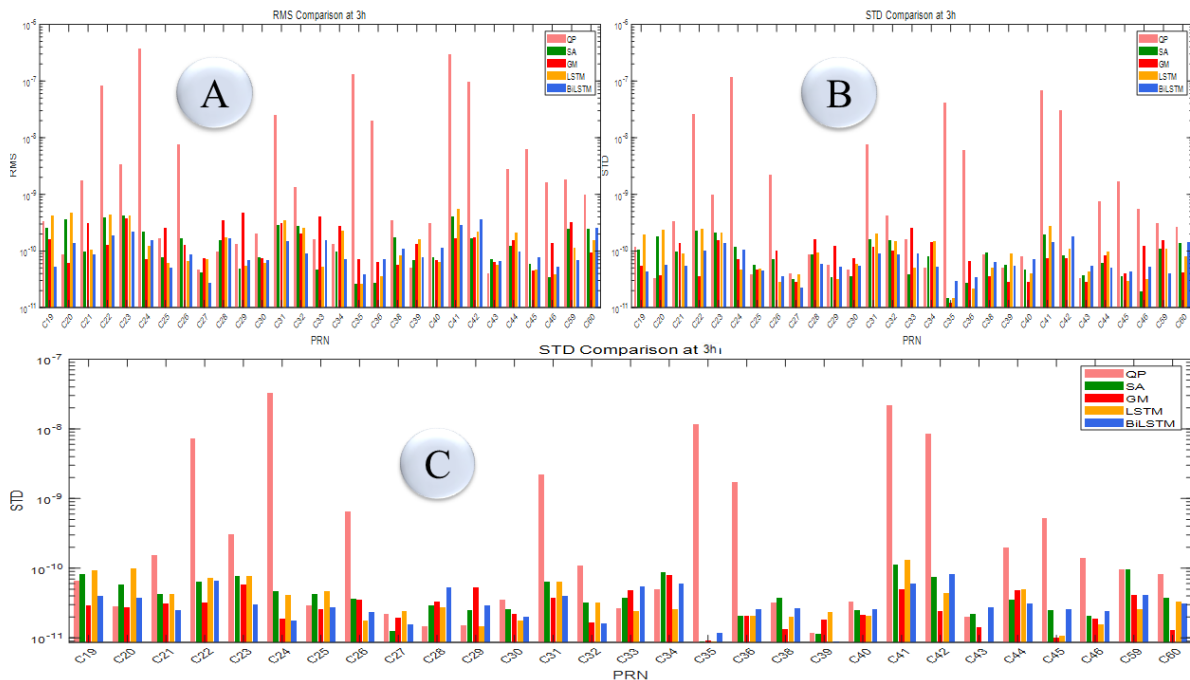


Figure 5: Performance comparison of different prediction models for 3-hour BeiDou satellite clock bias forecasting: (A) RMSE comparison, (B) Standard deviation (STD) comparison, and (C) Error range comparison.

C. Single-Day 6-Hour Clock Bias Prediction

To further test the ability of these models in predicting satellite clock bias of BEIDOU in the short duration of the six-hour forecasting experiment was performed. With forecasting horizon equal to 6 h instead of 1 h there is more uncertainty involved, since the uncertainty of the forecasts propagates as the forecast period lengthens. Hence, this experiment provides a tougher environment for a testing-while-learning analysis of the models' robustness and ability to learn over time.

Table 4: RMSE comparison of different models for single-day 6-hour BeiDou satellite clock bias prediction (ns)

PRN	QP	SA	GM(1,1)	LSTM	BiLSTM
C19	0.335	0.360	0.530	0.556	0.061
C20	0.196	0.580	0.229	0.794	0.135
C21	2.301	0.268	0.415	0.233	0.076
C22	149.400	0.511	0.500	0.593	0.172
C23	1.639	0.785	0.639	0.785	0.449
C24	0.637	0.281	0.135	0.103	0.183
C25	0.228	0.077	0.376	0.066	0.174
C26	1.578	0.148	0.126	0.081	0.133
C27	0.086	0.100	0.151	0.091	0.039
C28	0.183	0.254	0.619	0.268	0.171
C29	0.108	0.131	0.934	0.194	0.132
C30	0.149	0.063	0.359	0.182	0.117
C31	3.582	0.432	0.350	0.571	0.207
C32	2.230	0.495	0.339	0.492	0.153
C33	0.206	0.038	0.741	0.047	0.174
C34	0.119	0.263	0.550	0.548	0.073
C35	2.626	0.092	0.062	0.092	0.154
C36	3.511	0.213	0.095	0.249	0.078

C38	0.394	0.202	0.311	0.108	0.119
C39	0.151	0.102	0.333	0.215	0.117
C40	0.386	0.228	0.069	0.160	0.253
C41	4.438	0.618	0.141	0.977	0.332
C42	1.218	0.132	0.154	0.456	0.534
C43	0.069	0.101	0.089	0.122	0.055
C44	4.523	0.250	0.380	0.428	0.201
C45	1.126	0.067	0.229	0.126	0.092
C46	2.908	0.082	0.376	0.036	0.044
C59	2.777	0.276	0.279	0.353	0.129
C60	1.205	0.326	0.385	0.149	0.317

The 6-hour prediction results show that the longer time the forecasting period is, the more difficult it is to predict it than the 1-hour period forecasting. There are also some satellites where the RMSE of the QP results is significantly higher than that of the others, such as C22, C31 and C44, suggesting less success in modelling the nonlinearity of satellite clock bias by using a polynomial. Even though the GM(1,1) model does better in some cases than QP, it has been demonstrated to have unstable performance, because it is incapable of modeling the complex temporal variations.

Periodicity of satellite clock bias sequences is included in the SA model to increase the prediction accuracy, but the accuracy of the SA is still critically dependent on the correct identification of periodic pattern. Historic clock bias observations can be used by LSTM model to learn the temporal dependency to achieve a better performance compared to traditional approaches. However, it is only rendered in one direction, so it's not possible to make the most of full temporal data.

The model optimized based on the BiLSTM can learn temporal features both forward and backward, making the prediction performance most stable. This two-way learning ability allows the model to obtain more complex temporal relations and nonlinear changes to reduce the RMSE of most BeiDou sat. The RMSE distribution analysis also demonstrates that BiLSTM exhibits a compact and stable error distribution compared to other models.

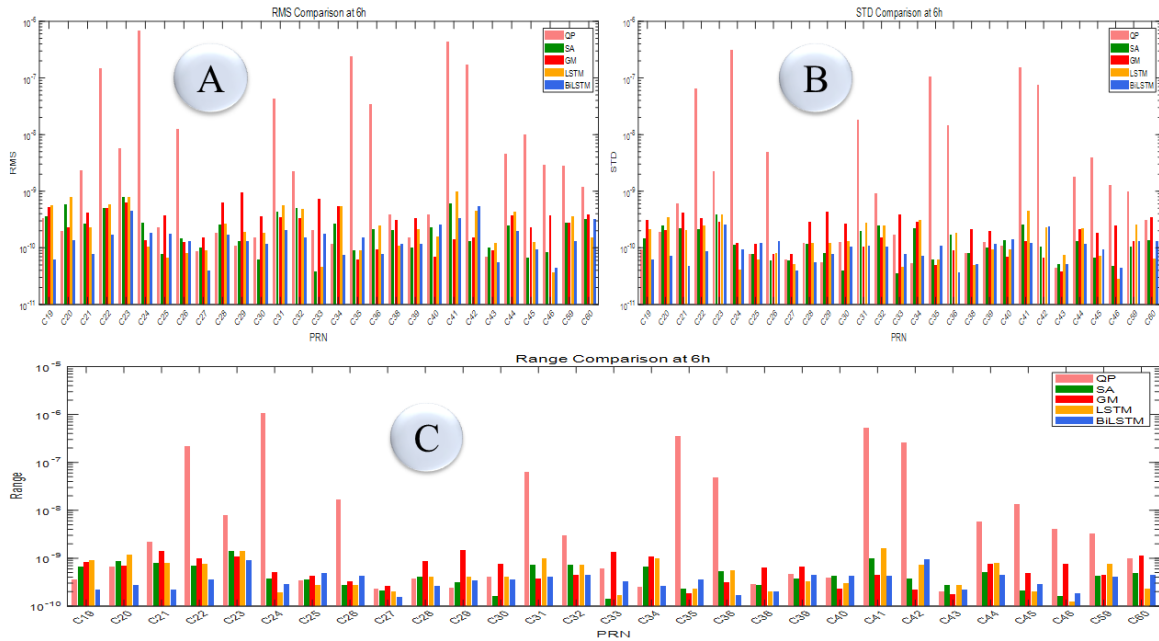


Figure 6: Performance comparison of different prediction models for 6-hour BeiDou satellite clock bias forecasting: (A) RMSE comparison, (B) Standard deviation (STD) comparison, and (C) Error range comparison.

The total error of various prediction models are shown under the condition of 6-hour clock bias prediction in figure 6. The Root Mean Square Error (RMSE) between QP model and SA model, GM (1,1) model, LSTM and BiLSTM model are shown in different BeiDou satellites, as shown in subfigure (A), the smaller the RMSE, the more accurate the prediction ability. Each subfigure (B) shows the standard deviation (STD) of prediction errors which is the stability and consistency of each model. Subfigure (C) shows the range of errors compared to the range of figures, indicating the sources of error and uncertainty. Globally, the optimized BiLSTM model outperforms the traditional models in terms of lower and more stable errors characteristics, reflecting that the optimized BiLSTM model can better account for the changes of satellite clock biases in nonlinear temporal structure.

D. Single-Day 12-Hour Clock Bias Prediction

The single-day 12-hour forecasting experiment was performed to evaluate the long-term short-term prediction capability of different models for BeiDou satellite clock bias. Compared with the 1-hour, 3-hour and 6-hour prediction scenarios, the 12-hour forecasting horizon introduces greater uncertainty because prediction errors accumulate over a longer period. Therefore, this experiment provides a more challenging evaluation environment to analyze the robustness and generalization ability of the proposed optimized BiLSTM model.

Table 5: RMSE comparison of different models for single-day 12-hour BeiDou satellite clock bias prediction (ns)

PRN	QP	SA	GM(1,1)	LSTM	BiLSTM
C19	0.662	0.856	1.095	1.220	0.107
C20	0.928	0.734	1.024	1.113	0.370
C21	3.812	0.738	1.634	0.689	0.104
C22	3.989	0.912	1.110	1.047	0.258
C23	1.816	1.756	1.424	1.687	0.860
C24	1.474	0.372	0.506	0.121	0.167

C25	0.318	0.218	0.645	0.088	0.384
C26	2.948	0.242	0.383	0.111	0.144
C27	0.195	0.108	0.124	0.204	0.099
C28	0.174	0.383	1.074	0.405	0.218
C29	0.206	0.190	1.949	0.348	0.101
C30	0.110	0.150	0.770	0.289	0.114
C31	0.924	0.686	0.375	0.996	0.343
C32	4.320	0.640	0.318	0.640	0.299
C33	0.555	0.126	1.735	0.099	0.219
C34	0.119	0.439	1.285	1.137	0.154
C35	5.181	0.236	0.133	0.255	0.335
C36	1.088	0.240	0.142	0.309	0.229
C38	0.915	0.483	0.543	0.367	0.757
C39	0.216	0.101	0.631	0.457	0.118
C40	0.844	0.343	0.068	0.268	0.296
C41	0.806	1.216	0.459	1.976	0.540
C42	1.412	0.107	0.135	0.884	1.101
C43	0.334	0.243	0.089	0.203	0.071
C44	0.988	0.442	0.608	0.737	0.360
C45	1.018	0.120	0.260	0.118	0.072
C46	2.333	0.166	1.040	0.032	0.049
C59	4.877	0.423	0.826	0.720	0.169
C60	1.574	0.461	1.677	0.156	0.420

The 12-hour forecasting results highlight the challenges in the satellite clock bias prediction with longer forecasting horizon. All models have greater uncertainty than shorter prediction intervals, as they deal with the fact that slight errors in the clock estimation can get compounded over time. The RMSEs obtained by the traditional QP model are higher for some satellites including C21, C22, C35, and C59, which reveals that the clock bias variations for these satellites are more complex with nonlinear behavior, and the polynomial model is inadequate to predict them. Although GM(1,1) model can give the better results for some satellites, there is still instability in GM(1,1) model because it can only use a

relatively small sample to model the dynamic temporal characteristics.

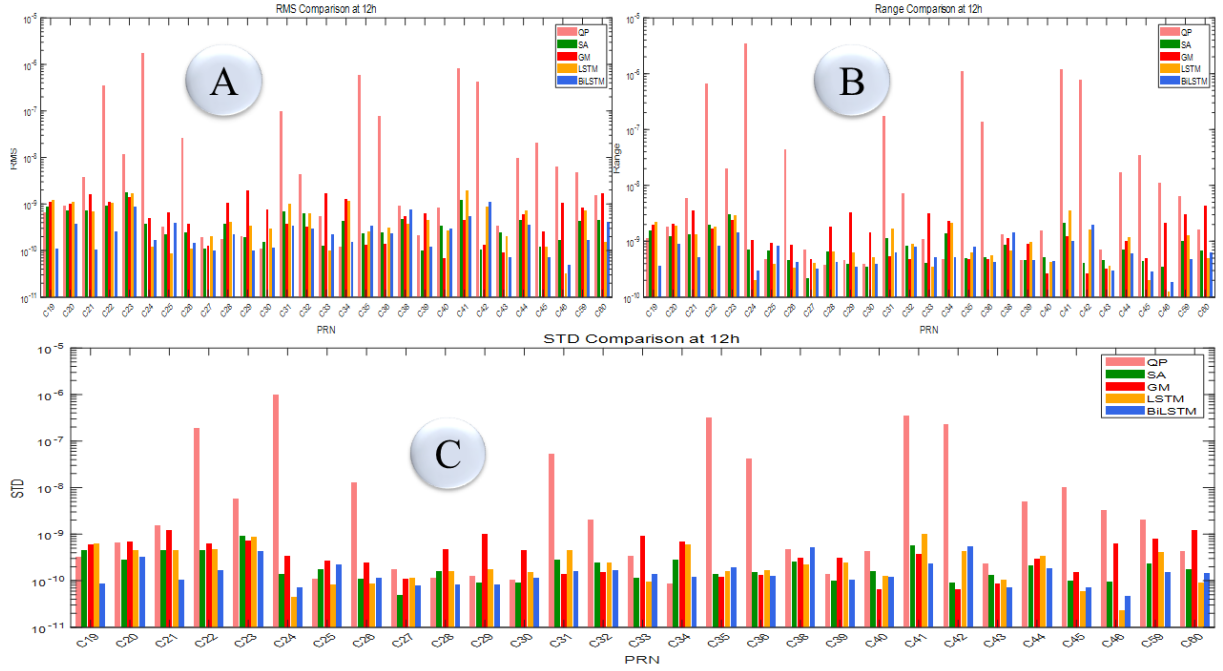


Figure 7: Performance comparison of different prediction models for 12-hour BeiDou satellite clock bias forecasting: (A) RMSE comparison, (B) Error range comparison, and (C) Standard deviation (STD) comparison.

This LSTM model is expected to outperform traditional methods as the ability of sequentially learning from previous clock bias data. Its structure is however unidirectional and is not able to capture full temporal information. Compared with other models, the optimized BiLSTM model has more stable performance in predicting the bias of the clock, because it considers the bidirectional temporal information during feature learning of the bias sequence of the clock. The results of the individual satellites demonstrate that there are different optimal models, but generally, BiLSTM achieves good and stable performance for most of the BeiDou satellites. BiLSTM offers better temporal representation ability which makes it more appropriate to tackle the task of satellite clock bias prediction under nonlinear and time-varying scenarios with a long horizon.

The whole error analysis for various prediction models of BeiDou satellite clock bias forecasting for 12-hour is shown in Figure 7. Subfigure (A) shows the Root Mean Square Error (RMSE) of QP model compared with SA model, GM (1,1) model, LSTM model, and BiLSTM model – a smaller RMSE value means that the model predicts the curve more accurately. Compare the error range, seen in (B) below, to variation and/or uncertainty of the prediction errors produced by the different models. The standard deviation (STD) analysis was shown in subfigure (C) and it was used to assess the stability and consistency of prediction performance. In fact, all the results show that the optimized BiLSTM model has lower and stable error characteristics than the traditional models, which indicate the better learning ability of the nonlinear temporal patterns of BeiDou satellite clock bias sequences.

E. Multi-Day Clock Bias Prediction

The multi-day prediction experiment was conducted to evaluate the long-term generalization capability of different prediction models for BeiDou satellite clock bias forecasting. Compared with single-day prediction experiments, multi-day forecasting provides a larger amount of historical information and allows the models to learn more complex temporal characteristics of satellite clock behavior.

Table 6: RMSE comparison of different models for multi-day BeiDou satellite clock bias prediction (ns)

PRN	QP	SA	GM(1,1)	LSTM	BiLSTM
C19	1.240	1.210	0.331	0.280	0.134
C20	1.220	1.390	0.442	0.150	0.200
C21	0.930	0.940	0.434	0.100	0.031
C22	0.345	0.620	0.373	0.420	0.310
C23	1.330	1.160	0.621	0.280	0.220
C24	2.240	1.630	1.736	0.360	0.120
C25	1.150	1.145	2.160	0.100	0.320
C26	1.630	1.620	1.258	0.150	0.047
C27	1.330	2.561	1.196	0.170	0.052
C28	2.540	0.752	0.370	0.435	0.062
C29	1.370	2.038	0.050	1.090	0.631
C30	1.420	1.435	0.410	0.435	0.442
C31	1.520	1.541	0.130	0.811	0.452
C32	0.190	0.561	0.330	0.047	0.552
C33	1.176	3.470	0.490	0.232	0.342
C34	1.132	2.140	0.130	0.358	0.441
C35	1.519	1.308	0.150	0.915	0.741
C36	1.321	2.220	0.090	0.279	0.311
C38	1.306	1.120	0.310	0.185	0.231
C39	2.240	2.233	1.300	0.148	0.563
C40	1.040	1.079	1.421	0.046	0.151
C41	2.340	1.350	0.940	0.130	0.045

C42	1.420	0.434	0.060	0.437	0.133
C43	1.210	1.620	0.130	0.840	0.230
C44	3.090	2.090	8.080	0.410	0.330
C45	3.140	1.340	0.290	0.560	0.157
C46	2.100	1.740	0.060	0.320	0.224
C59	0.923	0.988	0.110	1.170	0.551
C60	1.630	1.652	0.610	0.320	0.222

The multi-day prediction results show that the longer the prediction period, the more challenging the prediction accuracy occurs. Traditional models (e.g., QP, SA) for many satellites rely on mathematical assumptions, and have relatively high RMSE values, due to being unable to model complex nonlinear time evolution of satellite clock bias sequences. For some of satellites, the prediction stability of GM(1,1) model is fine, and the GM(1,1) model has better

performance for them. The LSTM model enables learning of sequential dependencies from historical clock bias observations and thus predicts the clock bias more effectively than traditional approaches.

The optimized BiLSTM model achieves competitive and stable performance in the multi-day forecasting scenario. BiLSTM is able to extract more complete temporal features from the sequences of satellite clock biases through this bidirectional learning about temporal. For a large number of satellites, the model gives the lower error in RMSE which signify that it has the ability to model nonlinear and long term temporal behavior of BeiDou satellite clock behavior. Through the multi-day experiment, the improved ability of the proposed BiLSTM framework in offering a stable prediction approach for a high precision timing application based on GNSS is confirmed, particularly in the presence of complex temporal variation of satellite clock bias.

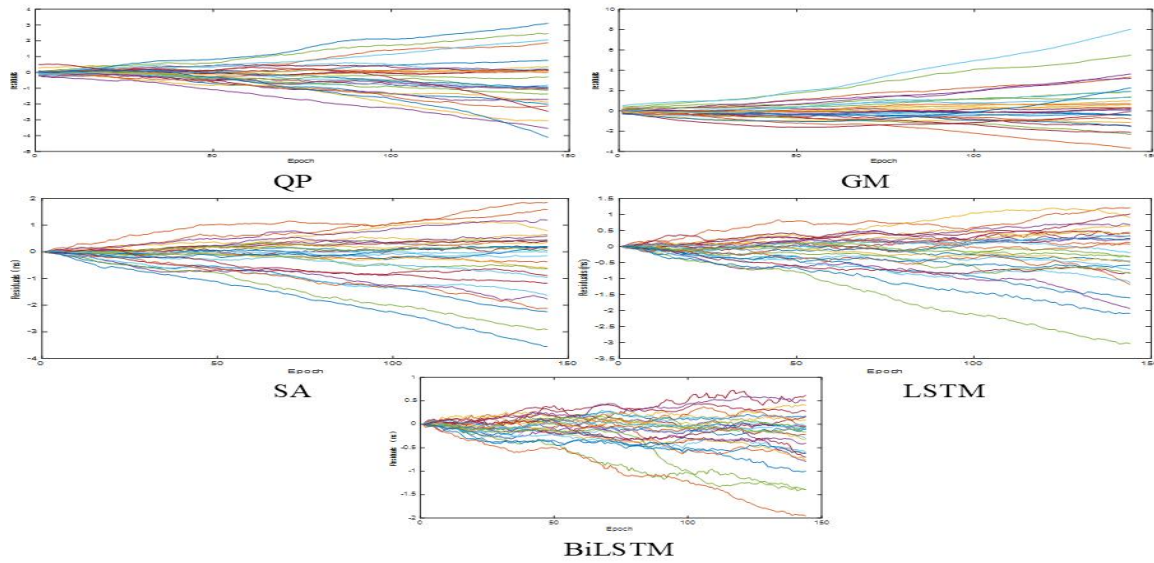


Figure 8: Residual curve of BDS multi-day forecast for different models.

Figure 8 presents the trend of prediction error of various models, such as QP, GM, SA, LSTM, and BiLSTM, in the case of the multi-day forecasting. The subplots depict the temporal change in the prediction error of a series of satellites. Based on the figure, one can see that the QP and GM models are widely dispersed in terms of the error curves as most of the trajectories become widely dispersed with time. This denotes that such conventional models generate poor and irregular forecasts, particularly in long term forecasting. SA model exhibits a better behavior than QP and GM, although they still exhibit a significant variation between sequences.

V. DISCUSSION OF KEY FINDINGS

Using the overall experiment assessment, which was conducted in this paper, a few significant conclusions may be made concerning the performance of various prediction models of satellite clock bias. The results on the other hand show that the deep learning models are of significant value compared to the traditional models in predicting the presence of the satellite clocks biasing effect. It is found that LSTM and BiLSTM, both of them have reduced value of RMSE in comparison to other models such as Quadratic Polynomial

(QP) model and GM (1,1) model. It is to indicate that using deep learning methods is more efficient in describing the nonlinearity, time dependency and complex nature of data on satellite clock bias. However, the existing models are based on rather simplistic assumptions, and thus, cannot be used to model dynamic variations, particularly if the horizons of prediction are long [29].

Secondly, the BiLSTM model is the most accurate prediction model of all the models considered. It provides always the minimum RMSE values for different satellite and forecasting period namely 1 hour and 3-hour, 6 hour and 12-hour forecasting. One plausible reason for this improvement in the BiLSTM is that it is a bidirectional model which is able to learn from both past and future time. It can learn from many more time lags with this model because learning works on both sides when the model learns from past events, it will learn underlying patterns of the data and predictive reliability as well as predictive accuracy will improve. Thirdly, we notice that the longer the period of forecast the less accurate is the forecast [30]. They all indicate that there is an increasing trend in the values of RMSE as the forecasts move to short-term (1-hour) to medium-term (6-hour) and long-term (12-hour) forecasts. This is likely to happen because of

uncertainty building up and error propagation with time. However, the overall trend does not mean that the BiLSTM model will not be susceptible to erratic growth of errors when compared to other models, indicating the model's robustness and capacity to mitigate such scenarios involving long term relationships [31]. The second important conclusion is that the BiLSTM model performs comparable with different satellites and setting up the atomic clock. The data set includes satellites with various types of orbits (MEO, IGSO and GEO) and a variety of satellite clock technologies (rubidium and hydrogen clocks) as well as different stability properties of the clocks. The BiLSTM model, although having such differences, achieves consistent and consistent prediction levels with high levels of generalization. It provides evidence that the model can be readily used to fit a range of form and patterns of data, and not necessarily changed manually [32].

Moreover, the use of Periodic (SA) model points out the relevance of describing cyclic patterns of the satellite clock biases. Although the performances of the deep learning-based methods are superior, SA model outperforms purely area based linear model using pure polynomials at capturing complicated nonlinear behaviors. Lastly, the experiments of multi-day predictions also confirm that the proposed approach is effective. BiLSTM model is also effective in single-day predictions as well as in instances where it is trained using long datasets. This means that the model will be backed by larger data size and in the future, it can learn larger patterns. The successes in the multi-day scenarios lend credibility to the accuracy of BiLSTM to actual GNSS in the real-world [33]. From a general perspective, the experiment results in this paper could assist to verify that the performance of the BiLSTM model to predict Satellite Clocks bias is very powerful and effective [34]. Results suggest that weaknesses exist within the classical models and that deep learning models and methods are highly beneficial in the context of time series modeling of complex GNSS data, particularly the bidirectional models [35].

VI. CONCLUSION

The high precision positioning applications of Global Navigation Satellite Systems have made improving the positioning accuracy and reliability through accurate satellite clock bias prediction to be critical. Based on this, optimized framework of Bidirectional Long Short-Term Memory (BiLSTM) network was proposed for short term prediction of BeiDou satellite clock bias in this study. Due to insufficient capturing abilities of traditional mathematical prediction model and traditional deep learning method on nonlinear and time-dependent characteristics of satellite clock bias sequences, the proposed approach was designed to overcome it. Using high accuracy BeiDou satellite clock bias data, a comprehensive assessment was carried out; the proposed BiLSTM model and Quadratic Polynomial (QP), Periodic (SA), Grey Model GM (1,1), conventional Long Short-Term Memory (LSTM) models were used for the assessment. A series of prediction experiments of 1-hour, 3-hour, 6-hour, 12-hour and multiple days were conducted to examine the prediction performance and stability of each model.

In comparison, the experimental results showed that the deep learning approaches exhibited in general a better

prediction capability than the traditional ones based on mathematical equations, because they were able to capture complex temporal dependencies. By employing a bidirectional temporal feature learning technique in the optimized BiLSTM model, the satisfactory and outstanding prediction performance was demonstrated in majority of the forecasting scenarios throughout the performance period. The model enhanced the reliability of clock bias forecast under various BeiDou satellite conditions and future looking directions and could successfully describe nonlinear variations of clock bias sequences of the BeiDou satellites nonlinearly. Based on the results of this study, one can conclude that the optimized BiLSTM model has great potential to deliver an efficient solution in the satellite clock bias prediction and can be used in high-precision GNSS timing and navigation systems. The proposed approach can help to give good positioning performance and system reliability by giving accurate clock prediction of satellite. In the future further research activities will target to improve the prediction framework by including more satellite-related factors, the development of adaptive learning solutions, considering adaptive behavior of the clocks, and finding lightweight DL architectures for real-time GNSS applications.

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